

I'm not a robot 
reCAPTCHA

Open

$$\Rightarrow \frac{\pi}{6} = \frac{\sqrt{3}}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{9}\right)^n \frac{1}{2n+1}$$

$$\Rightarrow \pi = 2\sqrt{3} \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)3^n}$$

This gives a series approximation for π .

**AP® CALCULUS BC
2003 SCORING GUIDELINES**

Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .

(c) At what time t is the coffeepot empty?

(a) $V = 25\pi h$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} = -5\pi\sqrt{h}$$

$$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$$

3 : $\begin{cases} 1 : \frac{dV}{dt} = -5\pi\sqrt{h} \\ 1 : \text{computes } \frac{dV}{dt} \\ 1 : \text{shows result} \end{cases}$

(b) $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$$\frac{1}{\sqrt{h}} dh = -\frac{1}{5} dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

$$2\sqrt{17} = 0 + C$$

$$h = \left(-\frac{1}{10}t + \sqrt{17} \right)^2$$

5 : $\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } h = 17 \\ \text{when } t = 0 \\ 1 : \text{solves for } h \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(c) $\left(-\frac{1}{10}t + \sqrt{17} \right)^2 = 0$

1 : answer

$$t = 10\sqrt{17}$$

Copyright © 2003 by College Entrance Examination Board. All rights reserved.
Available at apcentral.collegeboard.com.

6

Digitized by srujanika@gmail.com

Examination V

x	0	4	8	12	16
$g(x)$	28	27	24	19	12
$g'(x)$	0	-1	-2	-3	-4

3. Let $g(x)$ be a differentiable function defined on the interval $0 \leq x \leq 16$. Some values of $g(x)$ and its derivative $g'(x)$ are given in the table above. Which of the following is the x -intercept of the line tangent to the graph of $g(x)$ and parallel to the segment connecting the endpoints of $g(x)T$?
(A) $(23,0)$
(B) $(31,0)$
(C) $(-23,0)$
(D) $(-31,0)$

Answer _____

6. What is y^2 , if $\sin y = y + 2x^2$?

(A) $\frac{y}{1+\cos y}$ (B) $\frac{2\sin y}{(1+\cos y)^2}$ (C) $\frac{25\cos y}{(1+\cos y)^2}$ (D) $\frac{25\sin y}{(1+\cos y)^2}$

25. Find dy/dx by implicit differentiation. $x^2 + 7x + 13xy - y^2 = 16$

 - $\frac{dy}{dx} = \frac{x+7+13y}{y-13x}$
 - $\frac{dy}{dx} = \frac{2x+7+13y}{2x-13y}$
 - $\frac{dy}{dx} = \frac{2x+7-13y}{2y-13x}$
 - $\frac{dy}{dx} = \frac{2x+7-13y}{2y-13x}$
 - $\frac{dy}{dx} = \frac{2x+7+13y}{2y-13x}$

26. Find dy/dx by implicit differentiation: $\sin x + 2 \cos 5y = 5$

 - $\frac{dy}{dx} = \frac{\cos x}{10 \sin 5y}$
 - $\frac{dy}{dx} = \frac{\cos x}{10 \sin 5y}$
 - $\frac{dy}{dx} = \frac{\cos x}{5 \sin 5y}$
 - $\frac{dy}{dx} = \frac{\cos x}{10 \cos 5y}$
 - $\frac{dy}{dx} = \frac{\cos x}{5 \cos 5y}$

